Lesson 31

FIND VOLUME OF CONES, CYLINDERS, AND SPHERES NY-8.G.9

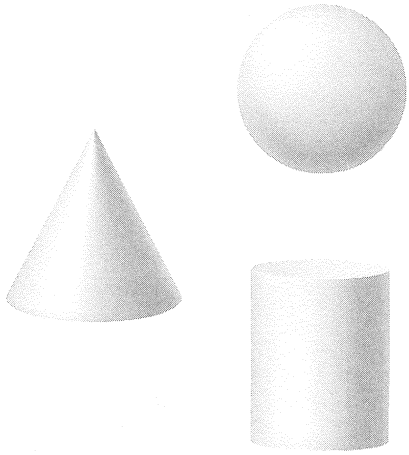
WORDS TO KNOW

radius

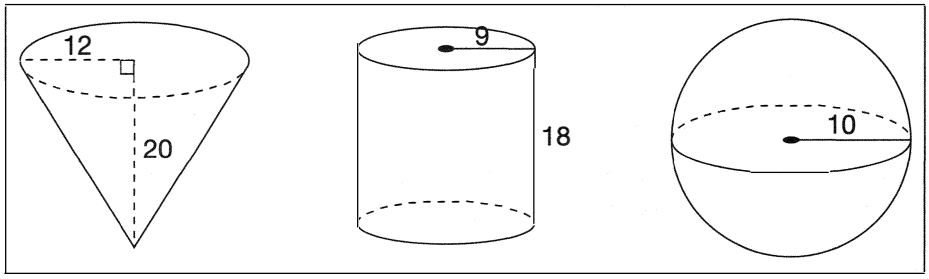
diameter

INTRODUCTION

Real-World Connection



Hassan's science teacher brought three containers to class: a cone-shaped container, a cylinder-shaped container, and a sphere-shaped container. His teacher gave the class a paper with drawings and information about the containers shown below, and asked all of the students to tell her how much space is in each container. How can Hassan find the volume of each container with the information he has?



Hassan can use formulas to find the volumes of the models. Let's practice the skills in the **Guided Instruction** and **Independent Practice**. At the end of the lesson, we will see how Hassan finds the volumes of the models!

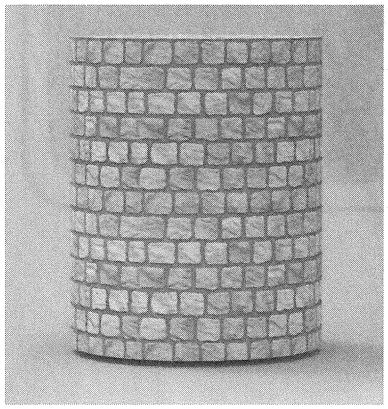
What I Am Going to Learn

- How to find the volume of cones, cylinders, and spheres

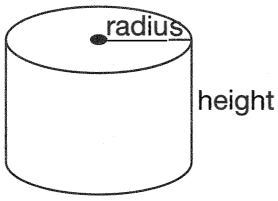
What I May Already Know

- I know and can identify cones, cylinders, and spheres.
- I can identify the diameter and radius of a circle.
- I can find the area of a circle.

Vocabulary in Action



- Use pi to find the volume of a cone, cylinder, and sphere. Pi (π) is irrational and goes on forever, but a good approximation is 3.14.
- The cone, cylinder, and sphere also involve circles somewhere, so their volumes use a radius measurement.
- The volume of a cylinder can be calculated using the formula $V = \pi r^2 h$, where r is the **radius** of the base, and h is the height of the cylinder. You can always find the radius if you are given the **diameter**, which is twice the radius.

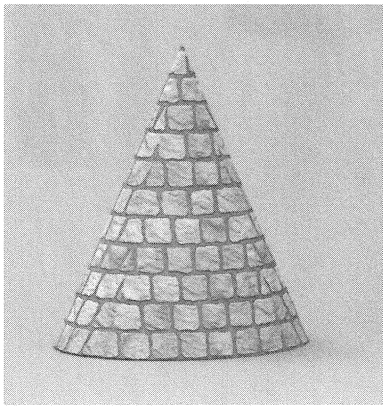


EXAMPLE

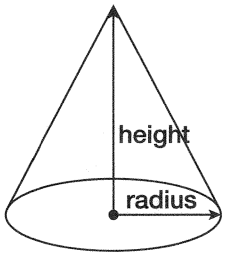
Find the volume of a cylinder with a radius of 2 cm and a height of 4 cm.

$V = \pi r^2 h$
$= \pi(2)^2 (4)$
$\approx (3.14)(4)(4)$
$\approx 50.2 \text{ cm}^3$

Remember that the volume is measured in cubic units.



The volume of a cone can be calculated using the formula $V = \frac{1}{3} \pi r^2 h$, where r is the radius and h is the height of the cone.



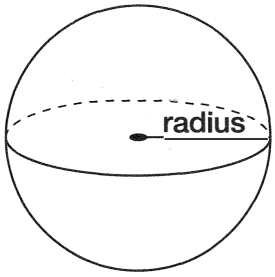
EXAMPLE

Find the volume of a cone with a radius of 2 cm and a height of 4 cm.

$V = \frac{1}{3} \pi r^2 h$
$= \frac{1}{3} \pi(2)^2 (4)$
$\approx \frac{1}{3} (3.14)(4)(4)$
$\approx 16.7 \text{ cm}^3$

Remember that the answer is an approximation because we are using an approximation of pi.

The volume of a sphere can be calculated using the formula $V = \frac{4}{3}\pi r^3$, where r is the radius.



EXAMPLE

Find the volume of a sphere with a radius of 6 cm.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(6)^3$

$\approx \frac{4}{3}(3.14)(216)$

$\approx 904.3 \text{ cm}^3$

GUIDED INSTRUCTION

1. A cylinder has a diameter of 10 feet and a height of 21 feet. What is the volume of the cylinder to the nearest tenth?

Step One Start with the formula for the volume of a cylinder.

$V = \pi r^2 h$

Step Two Identify the values you will use in the formula.

The radius is half the diameter.

$r = \frac{d}{2} = \frac{10}{2} = 5$

The height is 21 feet.

$h = 21$

Step Three Substitute the values into the formula.

$V = \pi(5)^2(21)$

$= \pi(25)(21)$

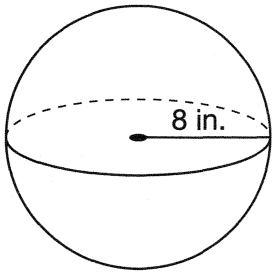
$\approx (3.14)(525)$

$\approx 1{,}648.5 \text{ ft}^3$

SHARE IT

What, if any, sphere-shaped piece of equipment do young people in your neighborhood use most often to play a game for fun?

2. What is the volume of the sphere shown below to the nearest tenth?



Step One Start with the formula for the volume of a sphere.

$V = \frac{4}{3}\pi r^3$

Step Two Identify the values you will use in the formula.

Spheres have one dimension: the radius.

This sphere has a radius of 8 in.

$r = 8$

Step Three Substitute the values into the formula.

$V \approx \frac{4}{3}(3.14)(8)^3 \approx 2{,}143.57$

Step Four Round the answer.

The volume is about 2,143.6 in.^{3}

3. Find the volume in cubic units of each figure described.

sphere: $r = 7$, $V \approx$ ____ cylinder: $r = 6$, $h = 8$, $V \approx$ ____

cone: $r = 9$, $h = 14$, $V \approx$ ____

Sphere, radius of 7 units:

$V = \frac{4}{3}\pi(7)^3$

$= \frac{4}{3}\pi($ [ ] $)$

$\approx \frac{4}{3}(3.14)(343)$

$\approx$ [ ] cubic units

Cylinder, radius of 6 units and a height of 8 units:

$V = \pi(6)^2 \times$ [ ]

$= \pi(36)(8)$

$\approx (3.14)(288)$

$\approx$ [ ] units^3

Cone, radius of 9 units and a height of 14 units:

$V = \frac{1}{3}\pi(9)^2(14)$

$= \frac{1}{3}\pi($ [ ] [ ] $)$

$\approx \frac{1}{3}(3.14)(1{,}134)$

$\approx$ [ ] units^3

TIPS AND HINTS

Be sure to take the square of the radius before multiplying by π.

Learning Together

In groups of 2 or 3, construct or find a cone, cylinder, or sphere. You can create your own or use an object such as a soup can, ice cream cone, or basketball. Use a ruler to measure the dimensions of your object. Then find the volume of your object. Remember to include the units. Now trade your object with other groups in the class. Find the dimensions and volume of each of the other objects. Compare found volumes between groups. When you have measured and calculated the volume of at least three objects, present your object, its dimensions, and its volume to the class.

How Am I Doing?

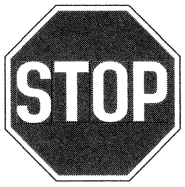
What questions do you have?

How do you find the volume of a cylinder? Of a cone? Of a sphere?

A sphere contains the largest volume possible for a given surface area.

Why do packaging designers not use spheres as containers for water?

Circle the sign that shows how you are doing with the skill.



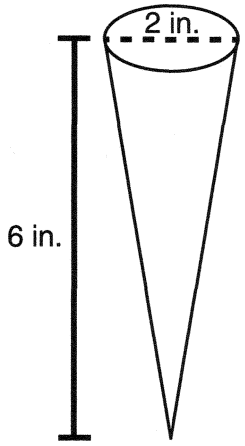
I am stuck.

I almost have it.

I understand the skill.

INDEPENDENT PRACTICE 1

1 A factory uses liners like the one shown below to collect stray particles.



TIPS AND HINTS

Remember that the formula for the volume of a cone is $\frac{1}{3}\pi r^2 h$.

What is the approximate volume, in cubic inches, of the liner?

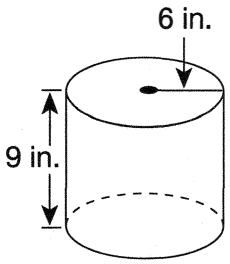
A 2

B 4

C 6

D 8

2 A cylinder is shown below.



TIPS AND HINTS

Remember that the formula for the volume of a cylinder is $\pi r^2 h$.

What is the approximate volume of the cylinder?

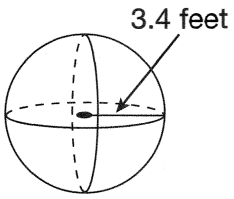
A 54 cubic inches

B 170 cubic inches

C 1,017 cubic inches

D 4,069 cubic feet

3 The sphere below is filled with concrete.



Rounded to the nearest whole number, what is the volume, in cubic feet, of 4.5 spheres?

A 165

B 218

C 740

D 14,995

TIPS AND HINTS

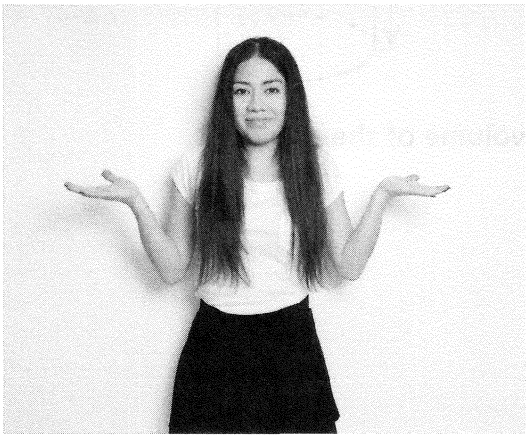
Remember that the formula for the volume of a sphere is $\frac{4}{3}\pi r^3$.

4 Carla needs a container that can hold at least 200 cubic units of liquid. She has a cylinder with a radius of 3 units and a height of 5 units. Can Carla use this container?

Explain your answer.

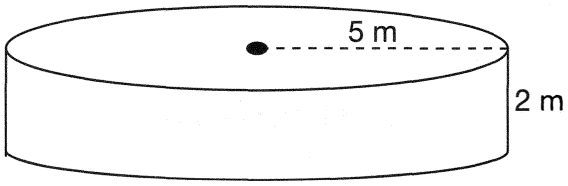
SKETCH IT

Draw the cylinder and mark the measurements so you can refer back to it as you work.



INDEPENDENT PRACTICE 2

1 A cylinder is shown below.



What is the volume of the cylinder to the nearest cubic meter?

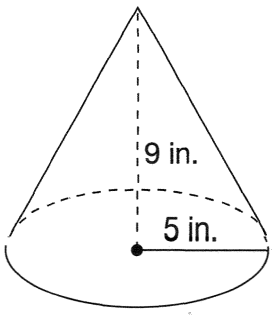
A 31

B 50

C 63

D 157

2 A cone is shown below.



What is the volume of the cone to the nearest cubic inch?

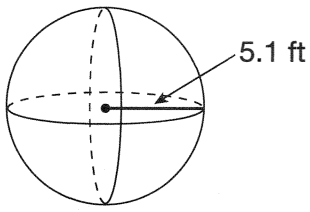
A 26

B 47

C 236

D 707

3 A spherical cage is shown below.



Rounded to the nearest whole number, what is the approximate volume, in cubic feet, of the cage? Use $\pi = 3.14$.

A 109

B 139

C 555

D 1,666

4 What is the volume, in units, of a sphere with a radius of 1 unit?

A 5

B 4

C 3

D 2

5 What is incorrect about the statement below?

"A cylinder with a base diameter of 12 inches and a height of 15 inches has a volume of 540 cubic inches."

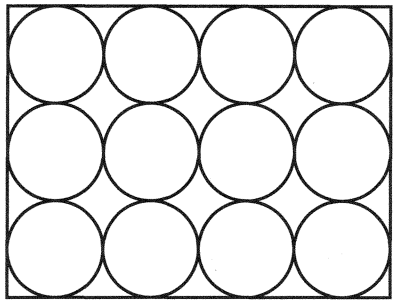
A The volume should be measured in square inches.

B The volume is actually 6,782.4 cubic inches.

C The volume is actually 1,695.6 cubic inches.

D There is nothing incorrect about the statement.

6 The image below shows 12 cans of juice arranged in 3 rows of 4. Each can has a radius of 4 centimeters and a height of 15 centimeters. The cans are packed in a box that has no extra space on the top, bottom, or sides.



How much space in the box, in cubic centimeters, will **not** be filled by a can in the box?

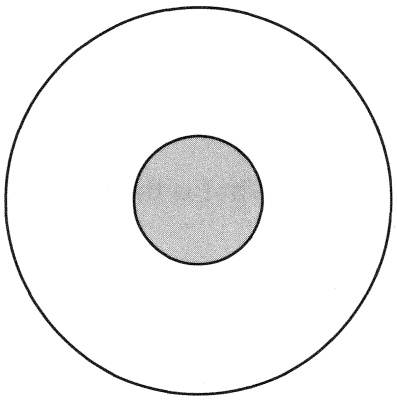
A 2,476.8

B 9,043.2

C 11,520

D 13,996.8

7 A larger cylinder has a central core within it. The diameter of the large cylinder is 12 centimeters and the diameter of the central core is 4 centimeters. The top of the cylinder is shown below.



If the cylinder has a height of 15 centimeters, what will be the volume of the cylinder if the central core is removed? Round your answer to the nearest cubic centimeter.

A 188

B 754

C 1,508

D 1,696

8 Soren found the volume of a sphere with a diameter of 20 using the steps below:

$V = \frac{4}{3}\pi(20)^3$

$= \frac{4}{3}\pi(8{,}000)$

$\approx \frac{4}{3}(3.14)(8{,}000)$

$\approx 31{,}400 \text{ units}^3$

What mistake did Soren make in his calculations?

Explain your answer.

_____

_____

Find the volume of the sphere to the nearest tenth.

Show your work.

Answer _____ units^3

9 A cylinder has a volume of 150 cubic units. What is the volume of a cone with the same radius and height as the cylinder?

Explain your answer.

_____

_____

_____

_____

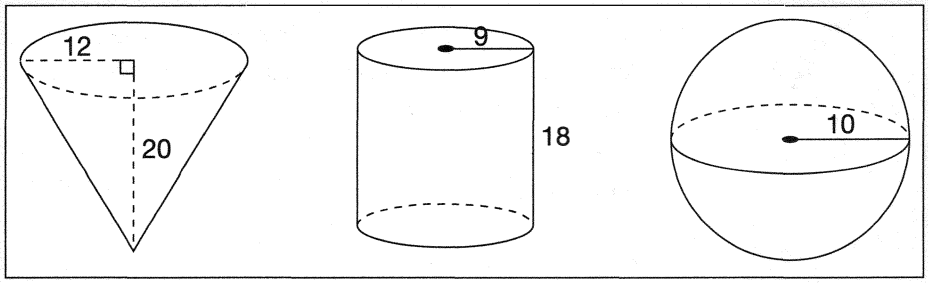
_____

EXIT TICKET

NY-8.G.9

Now that you have mastered finding the volume of cylinders, cones, and spheres, let's solve the problem in the **Real-World Connection.**

Hassan's science teacher brought three containers to class: a cone-shaped container, a cylinder-shaped container, and a sphere-shaped container. His teacher gave the class a paper with drawings and information about the containers shown below, and asked all of the students to tell her how much space is in each container. Find the volume of each container with the information given.



Cone: _____

Cylinder: _____

Sphere: _____